(7 pages) **Reg. No. :** ....

Code No. : 6373 **Sub. Code : ZMAM 25**

M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2022

Second Semester

Mathematics – Core

RESEARCH METHODOLOGY AND STATISTICS

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The one person who will almost certainly feature in the acknowledgement is

   (a) your father (b) your supervisor

   (c) your principal (d) your friend

2. Typically, abstracts are between ———— and ———— words in length.

   (a) 250 and 300 (b) 25 and 30

   (c) 2020 and 3000 (d) 5 and 10

3. Suppose a box contains 3 white balls and 2 black balls two balls are to be drawn successively at random and without replacement the probability that both balls drawn are black in

   (a) $\frac{1}{5}$ (b) $\frac{1}{10}$

   (c) $\frac{2}{5}$ (d) $\frac{1}{4}$

4. Let the joint p.d.f of X_1 and X_2 be

   $$f(x_1, x_2) = \begin{cases} x_1 + x_2, & 0 < x_1 < 1, 0 < x_2 < 1 \\ 0 & elsewhere \end{cases}$$

   The marginal probability density function $f(x_1)$ in $0 < x_1 < 1$ is

   (a) $x_1 + x_2$ (b) $x_1 + 1$

   (c) $x_1 \times x_2$ (d) x_1

5. If X has the p.d.f $f(x) = \begin{cases} \frac{1}{4} x e^{-x/2} & 0 < x < \infty \\ 0 & elsewhere \end{cases}$, then X is

   (a) $\chi^2(2)$ (b) $\chi^2(8)$

   (c) $\chi^2(-\frac{1}{2})$ (d) $\chi^2(4)$

6. If $M(t) = \rho^{3t+8t^2}$, then σ is

(a) 4 (b) 8

(c) 3 (d) 16

7. If $x_1 = \frac{1}{2}(y_1 + y_2)$, $x_2 = \frac{1}{2}(y_1 - y_2)$ then the value of J is

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$

(c) 2 (d) -2

8. If U and V are stochastically independent chi-square variables with r_1 and r_2 degrees of freedom, then which one of the following is an F-distribution

(a) $\frac{Ur_1}{Vr_2}$ (b) $\frac{U/r_1}{V/r_2}$

(c) $\sqrt{\frac{U/r_1}{V/r_2}}$ (d) $\frac{U/r_2}{V/r_1}$.

9. Let X_1 and X_2 be stochastically independent with normal distribution $n(6,1)$ and $n(7,1)$ respectively, then $X_1 - X_2$ is

(a) $n(-1,2)$ (b) $n(1,2)$

(c) $n(-1,1)$ (d) $n(0,1)$

10. Let $\overline{X}$ denote the mean of a random sample of size 128 from a gamma distribution with $\alpha = 2$ and $s = 4$ the variance of $\overline{X}$ is

(a) $\frac{8}{128}$ (b) $\frac{3}{128}$

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Write a model Acknowledgement for your research project.

Or

(b) Why do we have a literature review?

12. (a) Show that the random variables X_1 and X_2 with joint p.d.f $f(x_1, x_2) = 12x_1x_2(1 - x_2)$, $0 < x_1 < 1, 0 < x_2 < 1$, zero elsewhere, are stochastically independent.

Or

(b) Two dimensional random variable (x, y) has the joint p.d.f. $f(x, y) = 8xy, 0 < x < y < 1$, zero elsewhere. Find marginal and conditional distributions.

13. (a) Find the m.g.f of a gamma distribution.

Or

(b) If the random variable x is $N(\mu,\sigma^2)$ $\sigma^2 > 0$, prove that the random variable $W = (x-\mu)/\sigma$ is $N(0,1)$.

14. (a) Show that $S^2 = \frac{1}{n}\sum_{1}^{n}(x_i - \overline{x})^2 = \frac{1}{n}\sum_{1}^{n}x_i^2 - \overline{x}^2$, what $\overline{X} = \sum_{1}^{n} X_i / n$.

Or

(b) Let X have the p.d.f. $f(x) = 1$, $0 < x < 1$ zero elsewhere. Show that the random variable $y = -2\log x$ has a chi-square distribution with z degrees of freedom.

15. (a) If $X_1, X_2, \ldots X_n$ is a random sample from a distribution with m.g.f $M(t)$, show that the m.g.f of $\sum_{1}^{n} x_i$ and $\sum_{1}^{n} X_i / n$ are respectively, $[M(t)]^n$ and $[M(t/n)]^n$.

Or

(b) Let $\overline{X}$ denote the mean of a random sample of size 100 from a distribution test is $\chi^2(50)$. Compute an approximare value of $\Pr(49 < \overline{X} < 51)$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) What are the different components of a research project? Explain.

Or

(b) (i) Explain how referencing convention are applied according to their sources.

(ii) Explain plagiarism. How will you avoid it in your project report.

17. (a) Let X_1 and X_2 have joint p.d.f $f(x_1, x_2) = 2, 0 < x_1 < x_2 < 1$ zero elsewhere

Find (i) the marginal probability density function

(ii) Conditional p.d.f of x_1 given $x_2 = x_2, 0 < x_2 < 1$

(iii) Condition mean and conditional variance of x_1 given $x_1 = x_2$.

Or

(b) Let $F(x_1, x_2) = 2(x_1^2 x_2^3, 0 < x_1 < x_2 < 1$, zero elsewhere, be the joint p.d.f pg x_1 and x_2 find the conditional mean and variance of x_1. given $x_2 = x_2$, $0 < x_2 < 1$.

18. (a) Find the m.g.f of the normal distribution and hence find the mean and variance of a normal distribution

Or

(b) If the random variable X is $N(\mu,\sigma^2),\sigma^2 > 0$, prove that the random variable $V = (x - \mu^2 / \sigma^2 \, is \, \chi^2(1)$.

19. (a) Let X_1, X_2 be a random sample of size $n = 2$ from a standard normal distribution. Show that the marginal p.d.f of $y_1 = x_1 / x_2$ is that of a Cauchy distribution. You mat take $y_2 = x_2$.

Or

(b) Derive the F-distribution.

20. (a) Let $X_1, X_2, \ldots X_n$ denote a random sample of size $n \geq 2$ from a distribution that is $N(\mu,\sigma^2)$. Let S^2 be the variance of this random sample prove that $\frac{nS^2}{\sigma^2}$ is a χ^2- variable with parameter $n-1$.

Or

(b) State and prove a special case of the central limit theorem.